Course description: Mathematical models of physical systems are often described by systems of differential equations. Given a complete description of a physical system, we can solve the governing differential equations to predict the outcome of some physical measurements; this is called the forward problem. In an inverse problem, we use experimental measurements (observations) to reconstruct/infer the values of the parameters that characterize the physical system and appear in the governing differential equations as coefficients, boundary data, initial conditions, etc.

This course provides an introduction to inverse problems that are governed by partial differential equations (PDEs) and methods for their numerical solution. The focus will be on variational formulations, ill-posedness, regularization, discretization methods, and optimization algorithms for large-scale inverse problems. In addition to learning about the theoretical aspects of inverse problems, the students will develop numerical implementations to gain insight into the effect of measurement noise, regularization, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters. While our focus will be mainly on deterministic inverse problems, the course will also provide an introduction to the statistical formulations. Specifically, we will consider the Bayesian formulation of an inverse problem and highlight the connections between the deterministic and Bayesian perspectives of inverse problems. Model problems will be drawn from different areas of science and engineering, including image processing, continuum mechanics, and geosciences.

Prerequisites: Solid background in linear algebra (e.g., at the level of MA 405). Students should have also taken at least one course in differential equations (e.g., MA 341), and have some basic familiarity with PDEs. Background in numerical linear algebra (e.g., at the level of MA 580) is desirable. However, the required background in numerical methods will be covered as needed, albeit quickly. Also, the required background material on nonlinear optimization, variational methods, and basic probability will be covered throughout the course. Generally, a mathematically mature student will be able to acquire from the lectures, the necessary mathematical and computational background. For computational aspects of the course, students should be familiar with programming in either Matlab or Python. Contact me if you need more information about the prerequisites.

Intended topics: Introduction to inverse problems with PDEs; ill-posedness and regularization; variational methods, weak forms; PDE-constrained optimization problems; adjoint based gradient and Hessian computation; descent methods from nonlinear optimization, Newton-conjugate gradient method; Bayesian approach to inverse problems, and the relation to uncertainty quantification.

Target audience: The target audience of this course includes graduate students in applied mathematics and biomathematics with strong interests in numerical methods, numerical optimization, and parameter estimation. This course would also be of interest to graduate students in engineering who have strong interests in computational methods, model calibration, and parameter identification.